Lecture 8: Spans part 2 and Linear Dependence/Independence

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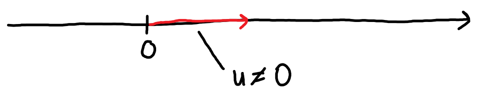
6.4 Subspaces of $\mathbb{R}, \mathbb{R}^2, \mathbb{R}^3$

a) $\mathbb{R}: \{0\}, \mathbb{R}$



If W is a subspace of $\mathbb{R}$ with $0 \neq u \in W$, then every $x \in \mathbb{R}$ is in W:

$x = \left(\frac{x}{u}\right) - u \in \text{span}(\{u\}) \subseteq W$

Hence, $W = \mathbb{R}$.

b) $\mathbb{R}^2: \{0\}, \text{lines through } 0, \mathbb{R}^2$

Either W$= \{0\}$, or $0 \neq u \in W$, **in which case** $span\{u\} \subseteq W$

Then, either W is line through 0, or $span\{u\} \not\ni v \in W$, **in which case** $span\{u, v\}$

Then, $W = \mathbb{R}^2$.

<u>NOTE</u>: The theorem from 6.3 also allows us to check if two spans are equal:

$span\{u_1, \dots, u_m\} = span\{v_1, \dots, v_n\}$

which is equivalent to:

$\{u_1, \dots, u_m\} \subseteq \text{span}\{v_1, \dots, v_n\}$ and $\{v_1, \dots, v_m\} \subseteq \text{span}\{u_1, \dots, u_n\}$

<u>Example</u>

$\text{span}\{(1,0), (0,1)\} = \text{span}\{(1,1), (1,-1)\}$

because:

$(1,0) = \frac{1}{2}(1,1) + \frac{1}{2}(1,-1), \quad (0,1) = \frac{1}{2}(1,1) - \frac{1}{2}(1,-1), \quad \text{so} \subseteq$

$(1,1) = (1,0) + (0,1), \quad (1,-1) = (1,0) - (0,1), \quad \text{so} \supseteq$

7. Linear (In-)Dependence

7.1 Difficulties with spans

Recall from chapter 6:

a) $span\{(1,0), (0,1)\} = span\{(1,1), (1,-1)\}$

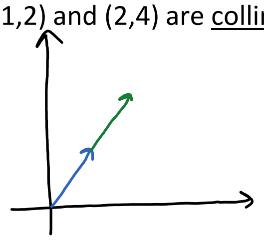
It isn't obvious if two subspaces are equal just by looking at their spanning sets.

b) $span\{(1,2), (2,4)\} = span\{(1,2)\}$

The number of elements in a spanning set does not necessarily tell you how "big" the subspace is.

<u>Problem</u>:

(1,2) and (2,4) are <u>collinear</u> ("on one line")



Similarly, $(1,0,0), (0,1,0), (1,1,0)$ are <u>coplanar</u> (on the same plane) and:

$span\{(1,0,0), (0,1,0), (1,1,0)\} = span\{(1,0,0), (0,1,0)\}$

"a plane through 0"

7.2 Collinearity in algebraic terms

Let u and v represent 2 collinear vectors. This means:

- ➢ $u = k * v$ for some $k \in \mathbb{R}$ <u>or</u> $v = k * u$ for some $k \in \mathbb{R}$

and:

- ➢ $\exists\ a, b \in \mathbb{R}$, not both 0, such that:
  $a * u + b * v = 0$

<u>Examples</u>

a) (1,2) and (2,4) are collinear because:
   $2(1,2) - 1(2,4) = (0,0)$
   in particular, linear combinations are not unique:
   $2(1,2) - 1(2,4) = 0(1,2) + 0(2,4)$
b) (3,1) and (0,0) are collinear because:
   $0(3,1) + 17(0,0) = (0,0)$

7.3 Coplanarity in algebraic terms

Let u, v, and w represent coplanar vectors.
This means:

- $u = kv + lw$ for some $k, l \in \mathbb{R}$
  <u>or</u>
  $v = ku + lw$ for some $k, l \in \mathbb{R}$
  <u>or</u>
  $w = ku + lv$ for some $k, l \in \mathbb{R}$
- $\exists\ a, b, c \in \mathbb{R}$, not all 0, such that:
  $au + bv + cw = 0$

<u>Example</u>

$(1,0,0), (0,1,0), (1,1,0)$ are coplanar because:
$1(1,0,0) + 1(0,1,0) - 1(1,1,0) = (0,0,0)$

7.4 Definition

V is a vector space, and $v_1, \dots, v_m \in$ V. Then $\{v_1, \dots, v_m\}$ is linearly independent (LD) if there are $a_1, \dots, a_m \in \mathbb{R}$, not all 0, such that $a_1 v_1 + \cdots + a_m v_m = 0$.